Low Rank Matrix Algebra for EM Integral Equations

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This ICEAA ***half day*** tutorial is intended as an introduction to low rank block matrix factorization methods for Method of Moments (MOM) frequency domain integral equations. Such methods and variations have enabled direct factorization LU solutions on inexpensive PC workstation class computers for problem sizes well over one million unknowns.

The original limitation of MOM formulations is the operations count tyranny of N^2 for matrix fill, N^3 for LU factorization and N^2 for solving each right-hand side (monostatic scattering can easily have 10,000 or more angles).

Problem size increase over the last 50 years is partly due to Moore's Law in computer technology. However, the main contributor in the last dozen years is the realization that with spatial grouping of unknowns, the interactions of such groups are of low rank. In particular, the introduction of the Adaptive Cross Approximation (ACA) led to our ability to compute sparse low rank blocks of the system matrix and of its LU block factors.

Course topics will include:

- What is low rank? How it is determined? How it is measured?
- How do we know if a matrix block is R_k
- Spatial grouping using the Cobblestone approach
- Adaptive Cross Approximation (ACA)
- SVD approximation is lower rank than ACA
    - Recompression: QR & SVD
- Thrill of R_k multiplication
- Agony of R_k addition
- ACA for computing R_k sums
- LU block ***factorization*** using R_k matrices
- RHS monostatic forcing function in R_k form
- LU ***solve*** using R_k matrices and the R_k form for the current solution J
- Computation of the polarization scattering matrix using the R_k forms of solutions J and row matrix R